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FINITE TIME THERMODYNAMIC EVALUATION OF IRREVERSIBLE ERICSSON AND STIRLING HEAT PUMP CYCLES

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Abstract

This communication presents the finite time thermodynamic evaluation of irreversible Ericsson and Stirling heat pump cycles, using ideal or real gas as the working substance alongwith the infinite heat capacities of heat source and heat sink reservoirs. The external irreversibilities in the two isothermal processes are due to finite temperature difference and the internal irreversibilities are due to the regenerative loss and the direct heat leak loss from the heat sink to the heat source and are responsible for the lower heating coefficient of performance.

The power input to the Ericsson and Stirling heat pumps is minimized for a given heating load and the heating coefficient of performance evaluated at optimum operating conditions. It is found that the power input is affected by the time of regenerative processes while, the heating coefficient of performance and then the heating load are affected by the regenerative heat loss as well as direct heat leak losses. It is also found that for perfect regeneration (ε_R=1.00) and without heat leak, the Ericsson and the Stirling heat pumps have the same performance characteristics similar to an endoreversible Carnot heat pump cycle for the same operating conditions.

INTRODUCTION

Finite time/finite temperature difference thermodynamics deals with the fact that there must be a finite temperature difference between the working substance and the reservoirs (with which it is in contact) for a finite amount of heat to be transferred in finite time. The novel work of Curzon-Ahlborn [1] remarked that the efficiency of an engine operating at maximum power is given by the formula [$\eta_m = 1-\sqrt{T_L/T_H}$], which is always smaller than the well known Carnot formula [$\eta_C = 1-T_L/T_H$] and agrees much better with the measured efficiencies of operating installations [2]. It is also desirable to have the corresponding results for the coefficient of performance (COP) of the refrigeration, airconditioning and heat pump systems. Leff and Teeters [3] have noted that the straight forward C-A calculations will not work for reversed Carnot cycle because there is no "Natural Maximum". Blanchard [4] has applied the Lagrangian method of undetermined multiplier to find out the COP of endoreversible Carnot heat pump operated at minimum power input for a given heating load. Further extension work on endoreversible Carnot and Brayton heat pump and RAC systems for different conditions have been carried out by earlier workers [5-16]. Chen [17] found the COP and power input of an irreversible Stirling refrigerator for a given cooling load applying the same method.

In this paper, we have applied the same Lagrangian method for irreversible Ericsson and Stirling heat pump cycles with infinite heat capacity of heat source/sink reservoirs, finite regeneration time, regenerative losses and direct heat leak losses from source to sink. We have obtained the optimum COP and power input at maximum heating load and also optimum power input at maximum COP for the given heating load. The effects of operating temperatures and the effectiveness of regenerator on the heat flow to and from the heat pump, the regenerative heat flow, power input, heating load and the COP of the Ericsson and Stirling heat pump cycles have been studied.

SYSTEM DESCRIPTION

It is well known that working substance in the Ericsson and Stirling cycles may be a gas, a magnetic material etc. and for different working substances, these cycles have different performance characteristics. When the working substance of these cycle is an ideal gas, the Ericsson cycle consists of two isothermal and two isobaric processes while the Stirling cycle consists of two isothermal and two isochoric processes as shown in Fig.1 with their T-s & p-v diagrams in Figs.2(a-d). These cycles approximate the expansion stroke of real cycle as an isothermal process 1-2, with an irreversible heat addition at constant temperature T_c. The heat addition to working fluid from the regenerator is modeled as isobaric (in Ericsson cycle) or isochoric (in Stirling cycle) process 2-3. The heat rejection compression stroke is modeled as an isothermal process 3-4 with irreversible heat rejection at constant temperature T_h. Finally, the heat rejection to the regenerator is modeled as isobaric (in Ericsson cycle)/isochoric (in Stirling cycle) process 4-1.

As mentioned earlier, the external heat transfer processes 1-2 and 3-4, in real Ericsson and Stirling heat pump cycles must occur in finite time. This inturn requires that these heat transfer processes must proceed through finite temperature difference and are therefore defined as being externally irreversible. During heat addition and rejection processes 1-2 and 3-4, the heat flow through a finite temperature difference T_L-T_c and T_h-T_H, from the heat source to heat pump and from heat pump to the heat sink respectively. If the regenerator is an ideal one, the heat addition during process 4-1 will be equal to the heat rejection during process 2-3, but an ideal regenerator requires an infinite area or infinite regeneration time and hence, it is desirable to consider a real regenerator. In addition, it is also desirable to consider the direct heat leak from sink to source. By considering all these factors, the heat pump is irreversible, in which the external irreversibility is due to finite temperature difference and internal irreversibility is due to regenerative and direct heat leak losses.

THERMODYNAMIC ANALYSIS

Let Q_h is the amount of heat rejected to the heat sink by the working fluid at constant temperature T_h and Q_c is the amount of heat absorbed by the working fluid from the heat source at temperature T_c then

$$Q_h = T_h \, \Delta s = k_1(T_h - T_H)t_H \tag{1}$$

$$Q_c = T_c \, \Delta s = k_2(T_L - T_c)t \tag{2}$$

where k_1 and k_2 are the thermal resistance between the isothermal heat rejection/addition processes and Δs is the entropy change during two isothermal processes and defined as:

$$\Delta s = nR\ln\lambda_r \tag{3}$$

where n is the number of moles of the working fluid, R is the universal gas constant and λ_r is the pressure/volume ratio of two regenerative processes i.e.

$$\lambda_r = \lambda_p = p_1/p_2, \text{ where } p_1>p_2 \text{ for Ericsson cycle}$$

$$\lambda_r = \lambda_v = v_2/v_1, \text{ where } v_2>v_1 \text{ for Stirling cycle}$$

The regenerative losses are also proportional to the temperature difference between the two processes, i.e.

$$\Delta Q_R = nC_f \, (1-\varepsilon_R) \, (T_h - T_c) \tag{4}$$

where C_f is the specific heat of the working fluid [$C_f = C_p$ for Ericsson cycle and $C_f = C_v$ for Stirling cycle] and λ_R is the regenerator effectiveness. In addition, the direct heat leak loss (Q_o) from sink to source is given [17] as:

$$Q_o = k_o \, (T_h - T_c)t_{cycle} \tag{5}$$

where k_o is the direct heat leak coefficient and t_{cycle} is the total cycle time. The net heat released to the sink and absorbed from the heat source will be

$$Q_H = Q_h - \Delta Q_R - Q_o \tag{6}$$

$$Q_L = Q_c - \Delta Q_R - Q_o \tag{7}$$

In addition, the influence of irreversibility of the finite heat transfer, the regenerative processes time is not negligible as compared to the time of two isothermal processes. These cycles have the different performance characteristics for different regeneration time and hence, we consider that the time of two regeneration processes is directly proportional to the temperature difference, as given by [17]

$$t_R = t_3 + t_4 = 2\alpha(T_h - T_c) \tag{8}$$

where α is the proportionality constant and independent of the temperature but dependent on the property of regenerative material. Thus, the cycle time will be

$$t_{cycle} = t_H + t_L + t_R \tag{9}$$

Such a model is useful because it includes all major irreversibilities.

OPTIMAL OPERATING CONDITIONS:

It is well known that the power input, heating/cooling load and the coefficient of performance (COP) are the three main parameters of the heat pump and RAC systems. Using the above equations, we have obtained the expression of these parameters for Ericsson and Stirling heat pump cycles as follows:

$$P = \frac{(Q_H - Q_L)}{t_{cycle}} = \frac{(x-1)}{\left[\frac{x}{k_1(xy-T_H)} + \frac{1}{k_2(T_L - y)} + b(x-1)\right]} \tag{10}$$

$$R_H = \frac{Q_H}{t_{cycle}} = \frac{x - a(x-1)}{\left[\frac{x}{k_1(xy-T_H)}\right]} - q \tag{11}$$

$$COP = \frac{R_H}{P} = \frac{x - a(x-1)}{(x-1)} - \frac{q}{(x-1)}\left[\frac{x}{k_1(xy-T_H)}\right] \tag{12}$$

where $x = T_h/T_c$; $y = T_c$; $b = 2\alpha/nR\ln\lambda_r$; $q = k_o(T_H - T_L)$ and $a = C_f(1-\varepsilon_R)/R\ln\lambda_r$

The purpose of any heat pump is to reject as much heat as possible to the heat sink with the expenditure of as little work as possible. This implies we should do our best to minimize power input for a given heating load.

For this we, introduce the Lagrangian L viz.

$$L = P + \lambda R_H \tag{13}$$

where λ is the Lagrangian multiplier thus, we have

$$L = \frac{(x-1) + \lambda[x - a(x-1)]}{\left[\frac{x}{k_1(xy-T_H)}\right]} - \lambda q \tag{14}$$

From Lagrangian-Euler equation $\frac{\partial L}{\partial y} = 0$ gives

$$x(T_L - y) = \sqrt{\frac{K_1}{K_2}}(xy - T_H) \tag{15}$$

From Eqs. (10-12) & (15) we have

$$P = \frac{K T_L (x-1)}{\left[\frac{x}{(x - T_H/T_L)} + B(x-1)\right]} \tag{16}$$

$$R_H = \frac{K\,T_L\,[x - a\,(x-1)]}{\left[\dfrac{x}{(x - T_H/T_L)}\right]} - q \tag{17}$$

$$COP = \frac{x - a\,(x-1)}{(x-1)} - \frac{E}{(x-1)}\left[\frac{x}{(x - T_H/T_L)}\right] \tag{18}$$

where $K = \dfrac{k_1 k_2}{(\sqrt{k_1} - \sqrt{k_2})^2}$; $B = b\,K\,T_L$; and $E = q/K\,T_L$

It is seen from Eqs.(16-18) that the power input, heating load and the COP are function of single variable 'x'. Thus, Eqs.(16-18) are used to determine the minimum power input and maximum COP for a given heating load and to discuss the other performance characteristics of both cycles.

I. Maximizing Heating Load:

It is seen from Eq.(17) that the heating load is not a monotonicly increasing function of power input unless both a and b are equal to zero, and there exists a maximum value for a given heating load. From Eq.(17) and its extremal condition we find that, when

$$x = \frac{(B \pm D)T_H/T_L}{(a + B - 1)} \tag{19}$$

the heating load attains its maximum

$$R_{H\,max} = K\,T_L \frac{\left[\dfrac{(B \pm D)T_H/T_L(1-a)}{(a+B-1)} + a\right]}{\dfrac{(B \pm D)}{(1 + D - a)} + B\left[\dfrac{(B \pm D)T_H/T_L}{(a+B-1)} - 1\right]} - q \tag{20}$$

The power input and the COP at maximum heating load are given as:

$$P(R_{H\,max}) = \frac{K\,T_L}{\dfrac{(B - D)}{(1 + D - a)}\left[\dfrac{(B \pm D)T_H/T_L}{(a+B-1)} - 1\right] + B} \tag{21}$$

$$COP(R_{H\,max}) = \frac{\left[\dfrac{(B \pm D)T_H/T_L}{(a+B-1)}\right]}{\left[\dfrac{(B \pm D)T_H/T_L}{(a+B-1)} - 1\right]} - a - E\left[\frac{(B \pm D)}{(1 \pm D - a)\left[\dfrac{(B \pm D)T_H/T_L}{(a+B-1)} - 1\right]} + B\right] \tag{22}$$

where

$$D = \sqrt{a\,(a + B - 1)\,T_L/T_H - B\,(a-1)}$$

Thus, from Eqs.(20-22) it may be seen that there are two different values for the power input for a given heating load where one is larger than $P_m\,(R_{H,\,max})$ and another is smaller than $P_m\,(R_{H,\,max})$. It is thus, clear that the power input smaller than $P_m\,(R_{H,\,max})$ is only minimum power input of these heat pump cycles for a given heating load.

II. Maximizing the Coefficient of Performance:

From Eq.(18) we can see that the heating COP is not monotonicly decreasing function of the heating load R_H unless $k_0 = 0$. From Eq.(18) and its extremal condition we find that when

$$x = \frac{(1 \pm U) T_H / T_L}{(1 - E)} \quad (23)$$

$$COP_{max} = \frac{(1 \pm U) T_H}{[(1 \pm U) T_H - (1 - E) T_L]} - a - E\left[\frac{(1 \pm U)(1 - E) T_L}{(E \pm U)[(1 \pm U) T_H - (1 - E) T_L]} + B\right] \quad (24)$$

In such case the heating load and the power input are, respectively determined by:

$$R_H(COP_{max}) = K T_L \frac{\left[\frac{(1 \pm U) T_H}{[(1 \pm U) T_H - (1 - E) T_L]}\right]}{\left[\frac{(1 \pm U)(1 - E) T_L}{(E \pm U)[(1 \pm U) T_H - (1 - E) T_L]} + B\right]} - q \quad (25)$$

$$P(COP_{max}) = \frac{K T_L}{\left[\frac{(1 - E)(1 \pm U) T_L}{[(1 \pm U) T_H - (1 - E) T_L]} + B\right]} \quad (26)$$

$$\text{where } U = \sqrt{E(E - 1) T_L / T_H + E} \quad (27)$$

Thus, it is seen from Eqs.(24-26), that there exists two values for each parameter in which one is above the optimal and another is below. Hence, the optimal operating regions are:

$$P(COP_{max}) \le P \le P(R_{H,max}) \quad (28)$$

$$COP_{max} \ge COP \ge COP(R_{H,max}) \quad (29)$$

$$R_H(COP_{max}) \le R_H \le R_{H,max} \quad (30)$$

According to Eqs.(28-30), we can determine the working fluid temperatures in two isothermal processes i.e.

$$T_{h,COP} \le T_h \le T_{h,RH} \quad (31)$$

$$T_{c,COP} \ge T_c \ge T_{c,RH} \quad (32)$$

and can be calculated from Eqs.(19) & (23) at both the conditions.

Special Cases**:**

1. When ε_R =1.00, i.e. the Ericsson and Stirling heat pumps possess the condition of perfect regeneration, although the time of regeneration processes is considered. In such case a = 0.0 and D=$\sqrt{B}$, from Eqs.(20-22) we have

$$R_{H\,max} = K T_L \left[\frac{\frac{(B \pm \sqrt{B}) T_H}{[(B \pm \sqrt{B}) T_H - (B - 1) T_L]}}{\frac{(B \pm \sqrt{B})(B - 1) T_L}{(1 \pm \sqrt{B})[(B \pm \sqrt{B}) T_H - (B - 1) T_L]} + B}\right] - q \quad (33)$$

$$P(R_{H\,max}) = \frac{K T_L}{\left[\frac{(B - 1)(B \pm \sqrt{B}) T_L}{(1 \pm \sqrt{B})[(B \pm \sqrt{B}) T_H - (B - 1) T_L]} + B\right]} \quad (34)$$

$$\mathrm{COP}(R_{H\max}) = \frac{(B\pm\sqrt{B})\,T_H}{[(B\pm\sqrt{B})T_H-(B-1)T_L]} - E\left[\frac{(B\pm\sqrt{B})\,(B-1)\,T_L}{(1\pm\sqrt{B})[(B\pm\sqrt{B})T_H-(B-1)T_L]} + B\right] \quad (35)$$

It can be seen from these that for perfect regeneration the Ericsson and Stirling heat pump have the same performance characteristics as that of endoreversible Carnot heat pump with direct heat leak loss is given by Eq.(7) and the regeneration time is given by Eq.(10). However, physically for finite regenerative time, ε_R should be less than unity. This shows that in the investigation of the Erisson and Stirling heat pump, it would be impossible to obtain new conclusions if the regenerator losses were not considered.

2. When α=0.0, i.e. the time of regeneration processes is negligible as compared to the time of two isothermal processes, although the regenerative losses are still considered. In such case b=B=0.0 and D=$\sqrt{\{a(a-1)T_L/T_H\}}$ and x has the value of $\sqrt{\{aT_H/(a-1)T_L\}}$
Thus, the maximum heating load will be:

$$R_{H,\max} = K\,\frac{\left[\sqrt{a\,T_L} - \sqrt{(a-1)\,T_H}\right]^2}{\sqrt{(a-1)}} - q \quad (36)$$

$$P(R_{H,\max}) = K\,\frac{[\sqrt{a\,T_H}-\sqrt{(a-1)\,T_L}\,][\sqrt{a\,T_L}-\sqrt{(a-1)}\,T_H]}{\sqrt{(a-1)}} \quad (37)$$

$$\mathrm{COP}(R_{H,\max}) = \frac{\left\lfloor\sqrt{a\,T_L} - \sqrt{(a-1)T_H}\right\rfloor}{[\sqrt{a}\,T_H-\sqrt{(a-1)}\,T_L]} - a - E_1\left[\frac{\sqrt{(a-1)}}{[\,(\sqrt{a\,T_L}-\sqrt{(a-1)\,T_H}\,]\,[\sqrt{a}\,T_H-\sqrt{(a-1)}\,T_L]}\right] \quad (38)$$

It can also be seen that the performance characteristics of Ericsson and Stirling heat pump cycles are different from that of a Carnot heat pump because the regenerative processes don't exist in the Carnot cycles.

3. When $t_R=\gamma(t_H + t_L)$, i.e. the time of two regeneration processes is directly proportional to the time of two isothermal processes. In this case we have:

$$R_{H,\max} = \frac{K'\;T_L[\sqrt{T_H} - a(\sqrt{T_H}-\sqrt{T_L})][\sqrt{T_H}-\sqrt{T_L}\,]}{\sqrt{T_H\,T_L}} - q \quad (39)$$

$$P(R_{H,\max}) = K'[(\sqrt{T_H}-\sqrt{T_L}\,]^2 \quad (40)$$

$$\mathrm{COP}(R_{H,\max}) = \frac{\sqrt{T_H}}{[\sqrt{T_H}-\sqrt{T_L}]} - a - E_2\,\frac{\sqrt{T_H\,T_L}}{[\sqrt{T_H} - \sqrt{T_L}\,]^2} \quad (41)$$

where K' = K/(1+γ) and E_2 = E/(1+γ)

4. When k_0=0.0 i.e. there is no direct heat leak loss from the sink to heat source, the performance characteristics are given as:

$$\mathrm{COP}(R_{H\max}) = \frac{T_L\,(a+B-1)}{[T_H\,(B\pm D) - (a+B-1)\,]} \quad (42)$$

Similarly the maximum COP is given by:

$$\mathrm{COP}_{\max} = \frac{T_H}{(T_H - T_L)} - a \quad (43)$$

In such case both the power input and heating load are equal to zero. Thus, it is clear that the COP of the Ericsson and Stirling heat pumps with non-zero heating load is always smaller than the values determined by Eq.(43).

5. When (i) k_0=t_R=0.0; (ii) ε_R =1.0, t_R=0.0 and (iii) ε_R=1.0, t_R=k_0=0.0 the Ericsson and Stirling heat pumps have the same expressions as that of endoreversible Carnot heat pump. Thus, the results may be obtained as done by earlier worker [4].

DISCUSSION OF RESULTS

In order to have a numerical appreciation of the results, we have considered the heat sink temperature (T_H) in the range 310-335K, the heat source temperature (T_L) in the range 270-295K, the effectiveness of the regenerator in the range 0.40-1.00 and the volume & pressure ratio are 2.5 and 2.64 respectively. We studied the effect of each of these parameter (while keeping others as constant) on heat transfer to and from the heat pump, the regenerative heat transfer, the power input, the heating load and the heating COP of both the heat pump cycles. We obtain the results at optimum heating load as well as at optimum COP and the discussion of results is given below:

Tables-1(a-b) show the effect of heat sink/source temperatures (T_H) on the heat transfer to and from the heat pump, the regenerative heat transfer, the working fluid temperatures, the power input, the heating load and the heating COP of the both cycles at maximum heating load and maximum COP condition respectively.

Effect of T_H:

It is seen from Table-1(a) that at maximum heating load condition as the heat sink temperature increases, the heat transfer from the heat pump, the regenerative heat transfer, the power input and the working fluid temperatures increase while the heat transfer to the heat pump, the heating load and the heating COP decrease. The effect of T_H is more pronounced for the regenerative heat transfer and less pronounced for the source side fluid temperature. Whereas from Table-1(b), it can be seen that the effect of T_H is slightly different i.e. the heating load increases while the source side temperature decreases.

Tables-2(a-b) show the effect of heat sink/source temperatures (T_L) on the heat transfer to and from the heat pump, the regenerative heat transfer, the working fluid temperatures, the power input, the heating load and the heating COP of the both cycles at maximum heating load and maximum COP condition respectively.

Effect of T_L:

Table-2(a) show that as the source temperature increases, the heat transfer to and from the heat pump, the heating load, the heating COP and the source side working fluid temperature increase while the regenerative heat transfer, the power input and the sink side working fluid temperature decrease. Also from Table-2(b), it is seen that the heating load decreases with increasing T_L. The effect of T_L is more pronounced for the COP and less pronounced for the sink side working fluid temperature. Thus, the source temperature should be as high as possible from point of view lesser the power input as well as for higher heating load and COP.

Tables-3(a-b) show the effect of the regenerator effectiveness, on the heat transfer to and from the heat pump, the regenerative heat transfer, the working fluid temperatures, the power input, the heating load and the heating COP of the both cycles at maximum heating load and maximum COP conditions respectively.

Effect of ε_R:

Table-3(a) shows that at maximum heating load condition as the regenerator effectiveness increases, the heat transfer to and from the heat pump, the regenerative heat transfer, the power input, the heating load, the heating COP and the sink working fluid temperature increase while the source side working fluid temperature decreases. Table-3(b) shows that at maximum COP condition, the working fluid temperatures and the power input remain constant while the other parameters increase. The effect of $_R$ is more pronounced for the regenerative heat transfer and less pronounced for the sink side working fluid temperature.

CONCLUSIONS

Finite time thermodynamics has been applied to obtain the performance characteristics of irreversible Stirling and Ericsson heat pump cycles, including the regenerative heat losses, finite regeneration time and the direct heat leak losses. It is found that source temperature and the regenerative effectiveness should be high enough for both the cycles at both conditions (i.e. at maximum heating load and maximum COP). The performance is better for perfect regeneration (ε_R=1.00) but it requires infinite time or infinite regenerator area. Hence, the present analysis is more general and provides a new theoretical basis for design, performance evaluation and improvements of both heat pumps and also the results are moreover same for the both cycles and may be applied to other cycles.

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Nomenclature

C- Heat capacitance (kW/K); COP- Coefficient of performance; P- Power (kW); p- Pressure (kPa); Q- Heat (kJ); R- Heating load; T- Temperature (K); t- Time (s); v- Volume (m^3)

Subscripts

c- Cold side/sink side; h,H- Hotside/heat source/heating; L- Heat sink; m- Minimum power/maximum heating load condition; R- Regenerator

Greak

ε - Effectiveness .

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Table-1(a): Effect of T_H on the performance of Ericsson and Stirling heat pump at maximum heating load. ($\varepsilon_R = 0.90$, $T_L = 290$ K, $v_2/v_1 = 2.0$ and $p_1/p_2 = 2.64$)

Ericsson Heat pump									Stirling heat pump							
T_H K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H KJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
310	86.52	69.14	56.35	0.85	4.14	4.87	333.38	270.99	61.45	50.39	35.86	0.62	3.35	5.40	329.45	273.84
313	87.08	68.82	59.21	0.87	4.02	4.64	336.57	271.02	61.84	50.16	37.88	0.63	3.23	5.12	332.61	273.86
315	87.45	68.60	61.11	0.88	3.95	4.50	338.70	271.03	62.10	50.00	39.23	0.64	3.16	4.95	334.71	273.87
317	87.82	68.38	63.02	0.89	3.88	4.37	340.83	271.05	62.36	49.84	40.58	0.65	3.09	4.78	336.82	273.89
320	88.38	68.06	65.88	0.90	3.77	4.19	344.02	271.07	62.75	49.61	42.60	0.66	3.00	4.56	339.97	273.91
322	88.75	67.84	67.79	0.91	3.71	4.08	346.15	271.09	63.01	49.45	43.95	0.66	2.93	4.42	342.08	273.92
324	89.12	67.62	69.69	0.92	3.64	3.97	348.27	271.11	63.27	49.29	45.30	0.67	2.87	4.29	344.18	273.93
326	89.49	67.40	71.60	0.93	3.58	3.87	350.40	271.12	63.53	49.13	46.65	0.68	2.82	4.16	346.29	273.95
328	89.86	67.18	73.51	0.93	3.52	3.77	352.53	271.14	63.78	48.98	48.00	0.68	2.76	4.04	348.40	273.96
330	90.23	66.97	75.41	0.94	3.46	3.67	354.65	271.15	64.04	48.82	49.35	0.69	2.71	3.93	350.50	273.98
332	90.60	66.75	77.32	0.95	3.40	3.59	356.78	271.17	64.30	48.66	50.70	0.69	2.65	3.82	352.61	273.99
335	91.16	66.42	80.18	0.96	3.32	3.46	359.97	271.19	64.69	48.43	52.72	0.70	2.58	3.67	355.76	274.01

Table-1(b): Effect of T_H on the performance of Ericsson and Stirling heat pump at maximum COP. ($\varepsilon_R = 0.90$, $T_L = 290$ K, $v_1/v_2 = 2.5$ and $p_1/p_2 = 2.64$)

Ericsson Heat pump									Stirling heat pump							
T_H K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H KJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
310	84.41	77.93	21.01	0.06	0.69	11.39	311.69	288.43	60.24	55.61	15.00	0.06	0.67	11.36	311.69	288.43
313	84.95	77.50	24.16	0.08	0.78	9.95	314.96	288.21	60.62	55.30	17.25	0.08	0.76	9.92	314.96	288.21
315	85.32	77.21	26.26	0.09	0.84	9.18	317.14	288.06	60.88	55.10	18.75	0.09	0.82	9.14	317.14	288.06
317	85.68	76.93	28.36	0.11	0.90	8.53	319.32	287.91	61.14	54.89	20.25	0.10	0.87	8.49	319.32	287.91
320	86.23	76.50	31.51	0.13	0.98	7.70	322.59	287.69	61.52	54.58	22.50	0.12	0.94	7.66	322.59	287.69
322	86.59	76.22	33.62	0.14	1.03	7.24	324.77	287.55	61.78	54.38	24.00	0.14	0.98	7.20	324.77	287.55
324	86.95	75.94	35.72	0.16	1.08	6.83	326.95	287.40	62.04	54.17	25.50	0.15	1.03	6.78	326.95	287.40
326	87.32	75.65	37.82	0.17	1.13	6.47	329.14	287.26	62.30	53.97	27.00	0.17	1.07	6.42	329.14	287.26
328	87.69	75.37	39.92	0.19	1.18	6.14	331.32	287.12	62.55	53.76	28.51	0.18	1.10	6.09	331.32	287.12
330	88.05	75.09	42.03	0.21	1.22	5.85	333.51	286.98	62.81	53.56	30.01	0.20	1.14	5.79	333.51	286.98
332	88.42	74.80	44.13	0.23	1.26	5.58	335.70	286.84	63.07	53.35	31.51	0.21	1.17	5.52	335.70	286.84
335	88.97	74.38	47.29	0.25	1.32	5.22	338.99	286.63	63.46	53.04	33.76	0.24	1.21	5.16	338.99	286.63

Table-2(a): Effect of T_L on the performance of Ericsson and Stirling heat pump at maximum heating load.
($\varepsilon_R = 0.90$, $T_H = 330$ K, $v_1/v_2 = 2.5$ and $p_1/p_2 = 2.64$)

Ericsson Heat pump									Stirling heat pump							
T_L K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
270	88.34	59.54	93.37	1.01	2.81	2.78	355.38	252.00	62.64	43.46	62.16	0.74	2.13	2.88	351.09	254.70
272	88.53	60.28	91.57	1.01	2.87	2.86	355.30	253.92	62.78	44.00	60.87	0.74	2.18	2.96	351.03	256.63
274	88.72	61.02	89.77	1.00	2.93	2.93	355.23	255.83	62.92	44.53	59.59	0.73	2.23	3.05	350.96	258.56
276	88.91	61.77	87.97	0.99	2.99	3.01	355.15	257.75	63.06	45.07	58.31	0.73	2.29	3.15	350.90	260.48
278	89.09	62.51	86.18	0.99	3.05	3.10	355.08	259.66	63.20	45.61	57.03	0.72	2.34	3.25	350.84	262.41
280	89.28	63.25	84.38	0.98	3.12	3.18	355.01	261.58	63.34	46.14	55.75	0.72	2.40	3.35	350.79	264.34
282	89.47	64.00	82.59	0.97	3.18	3.27	354.93	263.49	63.48	46.68	54.47	0.71	2.46	3.45	350.73	266.26
284	89.66	64.74	80.79	0.96	3.25	3.37	354.86	265.41	63.62	47.21	53.19	0.71	2.52	3.57	350.67	268.19
286	89.85	65.48	79.00	0.96	3.32	3.47	354.79	267.32	63.76	47.75	51.91	0.70	2.58	3.68	350.61	270.12
288	90.04	66.22	77.21	0.95	3.39	3.57	354.72	269.24	63.90	48.28	50.63	0.69	2.64	3.80	350.56	272.05
290	90.23	66.97	75.41	0.94	3.46	3.67	354.65	271.15	64.04	48.82	49.35	0.69	2.71	3.93	350.50	273.98
292	90.52	68.08	72.73	0.93	3.57	3.84	354.55	274.02	64.25	49.62	47.43	0.68	2.81	4.13	350.42	276.87
295	90.71	68.82	70.94	0.92	3.65	3.96	354.48	275.94	64.40	50.16	46.15	0.67	2.88	4.28	350.36	278.80

Table-2(b): Effect of T_L of the performance of Ericsson and Stirling heqt pump cycle on maximum COP.
($\varepsilon_R = 0.90$, $T_H = 330$ K, $v_2/v_1 = 2.0$ and $p_1/p_2 = 2.64$)

Ericsson Heat pump									Stirling heat pump							
T_L K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
270	86.16	66.70	63.08	0.41	1.50	3.70	335.49	265.65	61.44	47.54	45.04	0.36	1.31	3.62	335.49	265.65
272	86.35	67.54	60.97	0.39	1.48	3.85	335.29	267.78	61.57	48.14	43.54	0.35	1.31	3.77	335.29	267.78
274	86.54	68.38	58.87	0.36	1.46	4.01	335.08	269.90	61.71	48.74	42.03	0.33	1.30	3.93	335.08	269.90
276	86.73	69.22	56.76	0.34	1.44	4.19	334.88	272.03	61.85	49.34	40.53	0.31	1.29	4.11	334.88	272.03
278	86.91	70.05	54.65	0.32	1.42	4.37	334.68	274.17	61.98	49.94	39.02	0.30	1.27	4.29	334.68	274.17
280	87.10	70.89	52.55	0.30	1.39	4.57	334.48	276.30	62.12	50.55	37.52	0.28	1.26	4.50	334.48	276.30
282	87.29	71.73	50.44	0.28	1.36	4.78	334.28	278.43	62.26	51.15	36.02	0.26	1.24	4.71	334.28	278.43
284	87.48	72.57	48.34	0.27	1.33	5.01	334.09	280.57	62.40	51.75	34.51	0.25	1.22	4.95	334.09	280.57
286	87.67	73.41	46.23	0.25	1.30	5.27	333.89	282.70	62.53	52.35	33.01	0.23	1.20	5.20	333.89	282.70
288	87.86	74.25	44.13	0.23	1.26	5.54	333.70	284.84	62.67	52.95	31.51	0.21	1.17	5.48	333.70	284.84
290	88.05	75.09	42.03	0.21	1.22	5.85	333.51	286.98	62.81	53.56	30.01	0.20	1.14	5.79	333.51	286.98
292	88.34	76.35	38.87	0.18	1.15	6.36	333.23	290.19	63.02	54.46	27.76	0.17	1.09	6.31	333.23	290.19
295	88.53	77.19	36.77	0.16	1.11	6.75	333.04	292.33	63.16	55.06	26.25	0.16	1.05	6.70	333.04	292.33

Table-3(a): Effect of ε_R on the performance of Ericsson heat pumpcycle on maximum heating load.
(T_L = 290, T_H = 330 K, v_2/v_1 = 2.0 and p_1/p_2 = 2.64)

	Ericsson Heat pump								Stirling heat pump							
ε_R	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
0.40	54.64	35.39	27.74	0.75	1.93	2.58	346.16	277.06	40.09	27.07	18.77	0.57	1.57	2.74	344.09	278.59
0.50	60.66	40.66	36.02	0.79	2.21	2.79	347.72	275.94	44.27	30.83	24.22	0.60	1.78	2.97	345.30	277.70
0.60	67.20	46.42	44.90	0.83	2.50	3.00	349.35	274.78	48.75	34.87	29.99	0.62	2.00	3.21	346.54	276.79
0.70	74.29	52.71	54.41	0.87	2.81	3.22	351.05	273.59	53.53	39.21	36.09	0.65	2.22	3.44	347.83	275.86
0.75	78.05	56.06	59.41	0.89	2.96	3.33	351.93	272.99	56.03	41.49	39.27	0.66	2.34	3.56	348.48	275.40
0.80	81.96	59.55	64.57	0.91	3.13	3.45	352.82	272.38	58.62	43.86	42.54	0.67	2.46	3.68	349.15	274.92
0.85	86.02	63.18	69.91	0.92	3.29	3.56	353.73	271.77	61.29	46.30	45.90	0.68	2.58	3.81	349.82	274.45
0.90	90.23	66.97	75.41	0.94	3.46	3.67	354.65	271.15	64.04	48.82	49.35	0.69	2.71	3.93	350.50	273.98
0.95	94.61	70.90	81.10	0.96	3.63	3.79	355.59	270.53	66.88	51.42	52.88	0.70	2.83	4.05	351.19	273.50
1.00	99.14	75.00	86.96	0.97	3.80	3.91	356.55	269.90	69.80	54.11	56.51	0.71	2.96	4.18	351.89	273.02

Table-3(b): Effect of ε_R on the performance of Ericsson heat pump cycle on maximum COP.
(T_L = 290, T_H = 330 K, v_2/v_1 = 2.0 and p_1/p_2 = 2.64)

	Ericsson Heat pump								Stirling heat pump							
ε_R	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K	Q_H kJ	Q_L kJ	Q_R kJ	P_m kW	R_H kW	COP	T_h K	T_e K
0.40	64.70	51.74	18.68	0.21	0.84	4.05	333.51	286.98	46.14	36.88	13.34	0.20	0.78	3.99	333.51	286.98
0.50	69.37	56.41	23.35	0.21	0.92	4.41	333.51	286.98	49.48	40.22	16.67	0.20	0.85	4.35	333.51	286.98
0.60	74.04	61.08	28.02	0.21	0.99	4.77	333.51	286.98	52.81	43.55	20.01	0.20	0.93	4.71	333.51	286.98
0.70	78.71	65.75	32.69	0.21	1.07	5.13	333.51	286.98	56.14	46.89	23.34	0.20	1.00	5.07	333.51	286.98
0.75	81.05	68.08	35.02	0.21	1.11	5.31	333.51	286.98	57.81	48.55	25.01	0.20	1.03	5.25	333.51	286.98
0.80	83.38	70.42	37.36	0.21	1.14	5.49	333.51	286.98	59.48	50.22	26.67	0.20	1.07	5.43	333.51	286.98
0.85	85.72	72.75	39.69	0.21	1.18	5.67	333.51	286.98	61.15	51.89	28.34	0.20	1.10	5.61	333.51	286.98
0.90	88.05	75.09	42.03	0.21	1.22	5.85	333.51	286.98	62.81	53.56	30.01	0.20	1.14	5.79	333.51	286.98
0.95	90.39	77.42	44.36	0.21	1.26	6.03	333.51	286.98	64.48	55.22	31.67	0.20	1.17	5.97	333.51	286.98
1.00	92.72	79.76	46.70	0.21	1.29	6.21	333.51	286.98	66.15	56.89	33.34	0.20	1.21	6.15	333.51	286.98